# Generalized parton distributions in a meson cloud model

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We present a model calculation of the generalized parton distributions where the nucleon is described by a quark core surrounded by a mesonic cloud. In the one-meson approximation, we expand the Fock state of the physical nucleon in a series involving a bare nucleon and two-particle, meson-baryon, states. We discuss the role of the different Fock-state components of the nucleon by deriving a convolution formalism for the unpolarized generalized parton distributions, and showing predictions at different kinematics.

## 1. The meson-cloud model for the nucleon

The convolution model for the physical nucleon, where the bare nucleon is dressed by its virtual meson cloud, has a long and successful history in explaining properties such as form factors [ 1] and parton distributions [ 2]. In this paper it has been revisited and applied for the first time to study Generalized Parton Distributions (GPDs) that have recently been introduced and discussed in connection with Deeply Virtual Compton Scattering (DVCS) and hard exclusive meson production (for reviews, see Refs. [ 3, 4, 5, 6]).

The basic assumption of the meson-cloud model is that the state of the physical nucleon $\tilde{N}$ can be decomposed according to the meson-baryon Fock-state expansion as a superposition of a bare nucleon state and states containing virtual mesons associated with recoiling baryons. This state, with four-momentum $p_N^\mu = (p_N^-, p_N^+, \mathbf{p}_{N\perp}) \equiv (p_N^-, \tilde{p}_N)$ and helicity $\lambda$, is an eigenstate of the light-cone Hamiltonian

$$H_{LC} = \sum_{B,M} \Big[ H_0^B(q) + H_0^M(q) + H_I(N, BM) \Big] . \tag{1}$$

In Eq. (1), $H_0^B(q)$ stands for the effective-QCD Hamiltonian which governs the constituent-quark dynamics, and leads to the confinement of three quarks in a baryon state; analogously, $H_0^M(q)$ describes the quark interaction in a meson state, and $H_I(N, BM)$ is the nucleon-baryon-meson interaction, and the sum is over all the possible baryon and meson configurations in which the nucleon can virtually fluctuate. In the calculation presented here we use perturbative treatment of the meson effects. So we truncate the Fock space expansion of the nucleon state to the one-meson components, and expand the nucleon wavefunction in terms of the eigenstates of the bare Hamiltonian $H_0 \equiv H_0^B(q) + H_0^M(q)$. The corresponding state of the physical nucleon $|\tilde{N}\rangle$ can be written as

$$|\tilde{p}_N, \lambda; \tilde{N}\rangle \quad = \sqrt{Z}|\tilde{p}_N, \lambda; N\rangle + \sum_{B,M} \int \frac{\mathrm{d}y \mathrm{d}^2\mathbf{k}_\perp}{2(2\pi)^3} \frac{1}{\sqrt{y(1-y)}} \sum_{\lambda', \lambda''} \phi^{\lambda (N,BM)}_{\lambda' \lambda''}(y, \mathbf{k}_\perp)$$

$$\times\, |yp_N^+, \mathbf{k}_\perp + y\mathbf{p}_{N\perp}, \lambda'; B\rangle\, |(1-y)p_N^+, -\mathbf{k}_\perp + (1-y)\mathbf{p}_{N\perp}, \lambda''; M\rangle, \qquad (2)$$

where we introduced the function $\phi^{\lambda\,(N,BM)}_{\lambda'\lambda''}(y,\mathbf{k}_\perp)$ to define the probability amplitude for a nucleon with helicity $\lambda$ to fluctuate into a virtual $BM$ system with the baryon having helicity $\lambda'$, longitudinal momentum fraction $y$ and transverse momentum $\mathbf{k}_\perp$, and the meson having helicity $\lambda''$, longitudinal momentum fraction $1-y$ and transverse momentum $-\mathbf{k}_\perp$. Furthermore, in Eq. (2), the constant $Z$, giving the probability that the physical nucleon is a bare core state, ensures the correct normalization of the nucleon wave function:

$$\langle p'^+, \mathbf{p}'_\perp, \lambda'; H | p^+, \mathbf{p}_\perp \lambda; H\rangle = 2(2\pi)^3 p^+ \delta(p'^+ - p^+)\delta^{(2)}(\mathbf{p}'_\perp - \mathbf{p}_\perp)\delta_{\lambda\lambda'}. \qquad (3)$$

## 2. The unpolarized generalized parton distributions

In the definition of GPDs we choose a symmetric frame of reference where the virtual photon momentum $q^\mu$ and the average nucleon momentum $\bar p_N^\mu = \frac{1}{2}(p_N^\mu + p'^\mu_N)$ are collinear along the $z$ axis and in opposite directions. Furthermore, $Q^2 = -q^\mu q_\mu$ is the space-like virtuality that defines the scale of the process, $t = \Delta^2 = (p'^\mu_N - p^\mu_N)^2$ is the invariant transferred momentum square, and the skewness $\xi$ describes the longitudinal change of the nucleon momentum, $2\xi = -\Delta^+/\bar p_N^+$.

For each flavor $q$ the soft amplitude corresponding to unpolarized GPDs reads

$$F^q_{\lambda'_N\lambda_N}(\bar x, \xi, \mathbf{\Delta}_\perp) = \frac{1}{2\sqrt{1-\xi^2}} \int \frac{\mathrm{d}z^-}{2\pi}\, e^{i\bar x\, \bar p_N^+ y^-} \langle p'_N, \lambda'_N | \overline{\psi}(-\tfrac{1}{2}z)\, \gamma^+\, \psi(\tfrac{1}{2}z) | p_N, \lambda_N\rangle \Big|_{z^+=\mathbf{z}_\perp=0}, \qquad (4)$$

where $\bar x$ defines the fraction of the quark light-cone momentum ($\overline{k}^+ = \bar x\, \bar p_N^+$), $\lambda_N$ ($\lambda'_N$) is the helicity of the initial (final) nucleon, and the quark-quark correlation function is integrated along the light-cone distance $z^-$ at equal light-cone time ($y^+ = 0$) and zero transverse separation ($\mathbf{z}_\perp = 0$) between the quarks. The leading twist (twist-two) part of this amplitude can be parametrized in terms of the chiral-even helicity conserving GPD, $H^q(\bar x, \xi, \mathbf{\Delta}_\perp)$, and the helicity flipping GPD, $E^q(\bar x, \xi, \mathbf{\Delta}_\perp)$, for partons of flavor $q$.

In the following we will derive the convolution formulas for the GPDs in the three different regions corresponding to $\xi \le \bar x \le 1$, $-\xi \le \bar x \le \xi$, and $-1 \le \bar x \le -\xi$.

### 2.1. The region $\xi \le \bar x \le 1$

In this region the GPDs describe the emission of a quark from the nucleon with momentum fraction $\bar x + \xi$ and its reabsorption with momentum fraction $\bar x - \xi$. In the meson-cloud model, the virtual photon can hit either the bare nucleon $N$ or one of the higher Fock states. As a consequence, the DVCS amplitude can be written as the sum of two contributions

$$F^q_{\lambda'_N\lambda_N}(\bar x, \xi, \mathbf{\Delta}_\perp) = Z\, F^{q,bare}_{\lambda'_N\lambda_N}(\bar x, \xi, \mathbf{\Delta}_\perp) + \delta F^q_{\lambda'_N\lambda_N}(\bar x, \xi, \mathbf{\Delta}_\perp), \qquad (5)$$

where $F^{q,bare}$ is the contribution from the bare proton, described in terms of Fock states with three valence quarks, and $\delta F^q$ is the contribution from the $BM$ Fock components of the nucleon state, corresponding to five-parton configurations. The valence-quark contribution $F^{q,bare}_{\lambda'_N\lambda_N}$ can be calculated in the light-front overlap representation derived in Ref. [ 7], and applied to the case of $N = 3$ valence quarks in in Ref [ 8, 9], where one can also find the explicit expression in terms of bare-nucleon light-cone wave functions (LCWFs) derived in a constituent quark model. The $\delta F^q_{\lambda'_N\lambda_N}$ term in Eq. (5) can further be split into two contributions, with the active quark belonging either to the baryon ($\delta F^{q/BM}$) or to the meson ($\delta F^{q/MB}$), i.e.

$$\delta F^q_{\lambda'_N\lambda_N}(\bar{x},\xi,\mathbf{\Delta}_\perp) = \sum_{B,M}\left[\delta F^{q/BM}_{\lambda'_N\lambda_N}(\bar{x},\xi,\mathbf{\Delta}_\perp) + \delta F^{q/MB}_{\lambda'_N\lambda_N}(\bar{x},\xi,\mathbf{\Delta}_\perp)\right]. \tag{6}$$

The first term in Eq. (6) corresponds to the case when the baryon is taken out from the initial proton with a fraction $\bar{y}_B+\xi$ of the average plus-momentum $\bar{p}^+_N$, and after the interaction with the initial and final photons is reinserted back into the final proton with a fraction $\bar{y}_B-\xi$ of the average plus-momentum $\bar{p}^+_N$. The transverse momentum of the baryon is $\bar{\mathbf{p}}_{B\perp}-\mathbf{\Delta}_\perp/2$ before, and $\bar{\mathbf{p}}_{B\perp}+\mathbf{\Delta}_\perp/2$ after the scattering process. The meson substate is a spectator during the whole scattering process. As final result, the convolution formula for $\delta F^{q/BM}_{\lambda'_N\lambda_N}$ reads

$$\begin{aligned}\delta F^{q/BM}_{\lambda'_N\lambda_N}(\overline{x},\xi,\mathbf{\Delta}_\perp) \;=\; & \frac{1}{\sqrt{1-\xi^2}}\sum_M\sum_{\lambda,\lambda',\lambda''}\int_{\overline{x}}^1\frac{\mathrm{d}\overline{y}_B}{\overline{y}_B}\int\frac{\mathrm{d}^2\overline{\mathbf{p}}_{B\perp}}{2(2\pi)^3}F^{q/B}_{\lambda'\lambda}\left(\frac{\overline{x}}{\overline{y}_B},\frac{\xi}{\overline{y}_B},\mathbf{\Delta}_\perp\right)\\ & \times\,\phi^{\lambda_N\,(N,BM)}_{\lambda\lambda''}(\tilde{y}_B,\tilde{\mathbf{k}}_{B\perp})\,[\phi^{\lambda'_N\,(N,BM)}_{\lambda'\lambda''}(\hat{y}'_B,\hat{\mathbf{k}}_{B\perp})]^*,\end{aligned} \tag{7}$$

where $F^{q/B}_{\lambda'\lambda}$ is the scattering amplitude from the active baryon in the $BM$ component of the nucleon, and can be obtained in terms of the baryon LCWFs as explained in Ref. [ 13].

Analogously, we can derive the meson contribution to the scattering amplitude, corresponding to the case when the pion takes part to the interaction process while the baryon remains as a spectator. In such a case the role of the meson and baryon substates is interchanged with respect to the situation described before. The meson is taken out from the initial proton with a fraction $\overline{y}_M+\xi$ of the average plus-momentum $\overline{p}^+_N$, and after the interaction with the initial and final photons is reinserted back into the final proton with a fraction $\overline{y}_M-\xi$ of the average plus-momentum $\overline{p}^+_N$. The transverse momentum of the meson is $\overline{\mathbf{p}}_{M\perp}-\mathbf{\Delta}_\perp/2$ before, and $\overline{\mathbf{p}}_{M\perp}+\mathbf{\Delta}_\perp/2$ after the scattering process. Viceversa, the baryon substate is inert during the whole scattering process. Therefore the meson contribution to the $\delta F^q_{\lambda'_N\lambda_N}$ scattering amplitude is given by

$$\begin{aligned}\delta F^{q/MB}_{\lambda'_N\lambda_N}(\overline{x},\xi,\mathbf{\Delta}_\perp) \;=\; & \frac{1}{\sqrt{1-\xi^2}}\sum_B\sum_{\lambda,\lambda',\lambda''}\int_{\overline{x}}^1\frac{\mathrm{d}\overline{y}_M}{\overline{y}_M}\int\frac{\mathrm{d}^2\overline{\mathbf{p}}_{M\perp}}{2(2\pi)^3}F^{q/M}_{\lambda'\lambda}\left(\frac{\overline{x}}{\overline{y}_M},\frac{\xi}{\overline{y}_M},\mathbf{\Delta}_\perp\right)\\ & \times\,\phi^{\lambda_N\,(N,BM)}_{\lambda''\lambda}(1-\tilde{y}_M,-\tilde{\mathbf{k}}_{M\perp})\,[\phi^{\lambda'_N\,(N,BM)}_{\lambda''\lambda'}(1-\hat{y}'_M,-\hat{\mathbf{k}}_{M\perp})]^*,\end{aligned} \tag{8}$$

where $F^{q/M}_{\lambda'\lambda}$ is the scattering amplitude from the active meson in the $BM$ component of the nucleon, given explicitly in Ref. [ 13] in terms of meson LCWFs.

### 2.2. The region $-1\le\overline{x}\le-\xi$

In this region, the scattering amplitude describes the emission of an antiquark from the nucleon with momentum fraction $-(\overline{x}+\xi)$ and its reabsorption with momentum fraction

$-(\overline{x}-\xi)$. Therefore, the only nonvanishing contribution comes from the active antiquark in the meson substate of the $BM$ Fock component of the nucleon wavefunction, i.e.

$$F^{q}_{\lambda'_N\lambda_N}(\overline{x},\xi,\boldsymbol{\Delta}_\perp)=\delta F^{q/MB}_{\lambda'_N\lambda_N}(\overline{x},\xi,\boldsymbol{\Delta}_\perp), \tag{9}$$

where $\delta F^{q/MB}_{\lambda'_N\lambda_N}(\overline{x},\xi,\boldsymbol{\Delta}_\perp)$ is given by the same convolution formula (8), with the integration range over $\overline{y}_M$ between $-\overline{x}$ and 1, and with the LCWF overlap representation of $F^{q/M}_{\lambda'\lambda}$ in the range $-1\leq\overline{x}\leq-\xi$.

### 2.3. The region $-\xi\leq\overline{x}\leq\xi$

In this region, the scattering amplitude describes the emission of a quark-antiquark pair from the initial proton. In the Fock-state decomposition of the initial and final nucleons we have to consider only terms where the initial state has the same parton content as the final state plus an additional quark-antiquark pair. In the present meson-cloud model, the initial state is given by the five-parton component of the $|BM\rangle$ Fock state, while the final state is described by the three-valence quark configuration, multiplied by the normalization factor $\sqrt{Z}$. In principle, we can distinguish between two cases: *i*) the active quark belongs to the baryon substate, and the active antiquark is in the parton configuration of the meson substate of the initial nucleon; *ii*) both the active quark and antiquark belong to the meson substate of the initial nucleon and the baryon is a spectator during the scattering process. However, this last contribution is vanishing because it involves the overlap of two orthogonal states, i.e. the wave functions of the baryon in the initial state and of the bare nucleon in the final state. As a consequence, the only non vanishing contribution corresponds to the case *i*), and it can be explicitly derived in terms of the overlap between the five and three parton components of the LCWFs of the physical nucleon state as explained in details in Ref. [ 13].

## 3. Results

In this section we present results for the GPDs in the meson cloud model by restricting ourselves to consider only the pion-cloud contribution and disregarding the contributions from mesons of higher masses which are suppressed. As a consequence, the accompanying baryon in the $|B\pi\rangle$ component of the dressed proton is a nucleon or a $\Delta$. For the bare-hadron constituents of the nucleon state we use the LCWFs in the relativistic constituent model adopted in previous works to describe the valence contribution to GPDs both in the chiral-even and chiral-odd sector [ 8, 9, 10, 11, 12, 13]. The model assumption to describe the vertex functions $\phi^{\lambda(N,BM)}_{\lambda',\lambda''}$ are given in details in Ref. [ 13].

First let us study the forward limit, $\xi=0$, $t=0$. In this limit the scattering amplitude without nucleon helicity flip reduces to the ordinary parton distribution, and the convolution formulas for $H^q(\bar{x},0,0)$ coincides with the formulation of the meson cloud model for the ordinary parton distributions in deep inelastic processes [ 2]. In Fig. 1 the spin-averaged $H^q$ and the helicity-flip $E^q$ GPDs are plotted together with the separated contributions from the bare proton (dashed-dotted line), the baryon (dashed lines) and the meson (dotted lines) in the baryon-pion fluctuation. All these contributions add up incoherently to give the total result (full curves). The bare proton contribution is always

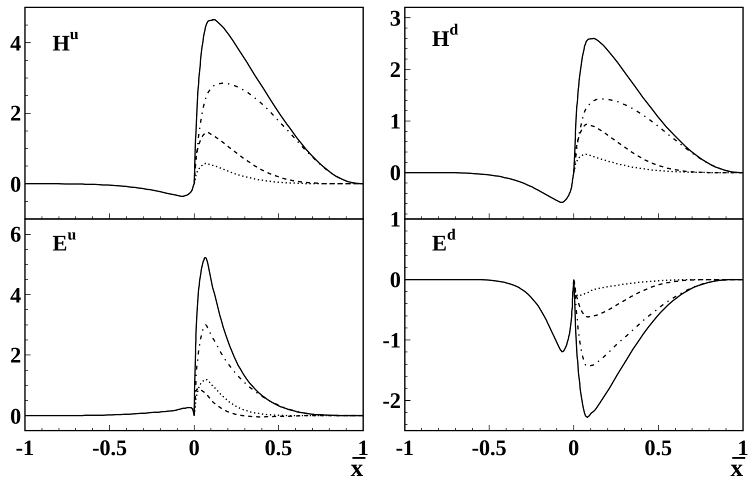


Figure 1. The different contributions to the spin-averaged ($H^q$, upper panels) and helicity-flip ($E^q$, lower panels) generalized parton distributions calculated in the meson-cloud model for flavours $u$ (left panels) and $d$ (right panels), at $\xi = 0$ and $t = 0$. Dashed lines: baryon contribution from the $|BM\rangle$ component. Dotted lines: meson contribution from the $|BM\rangle$ component. Dashed-dotted lines: contribution from the bare nucleon. Full curves: total result as a sum of the different contributions.

positive within its support ($0 \leq \overline{x} \leq 1$) with the exception of $E^d$ for which it is negative. The same behaviour characterizes the baryon contribution from the baryon-pion fluctuation that is also limited to the range $0 \leq \overline{x} \leq 1$, consistently with the assumption that the only active degrees of freedom for such a baryon are the valence quarks. The sea-quark contribution, extending all over the full range $-1 \leq \overline{x} \leq 1$, is determined by the antiquark residing in the meson of the baryon-pion fluctuation. The resulting effect of the pion cloud is thus to add a contribution for negative $\overline{x}$ and to increase the magnitude of the GPDs for positive $\overline{x}$ with respect to the case of the bare proton. In particular, for positive and small $\overline{x}$ the pion cloud contribution as a whole is comparable to that of the bare proton, confirming the important role of the sea at small $\overline{x}$. We also note the faster fall off of $E^q$ with respect to $H^q$ for $\overline{x} \rightarrow 1$, showing the decreasing role of the Melosh transform to generate angular momentum in $E^q$ with increasing quark momentum.

In all cases at $\overline{x} = 0$ the GPDs have a zero. This is due to fact that in the overlap integrals the various terms of the proton wavefunction are taken at one of their end points.

The forward limit of the first moment sum rule for the spin-averaged GPDs, is correctly fulfilled. For the helicity-flip GPDs, the first moment sum rule gives the quark anomalous magnetic moments, for which we find the values $\kappa^u = 1.14$ and $\kappa^d = -1.03$. Although these numbers are 5% and 10% off the experimental values for the up and down quarks, respectively, we found that the contribution of the pion cloud gives a substantial improvement with respect to the results obtained in the model with only valence quarks.

Going beyond the forward limit, we show in Fig. 2 the results for the GPDs at $\xi = 0$

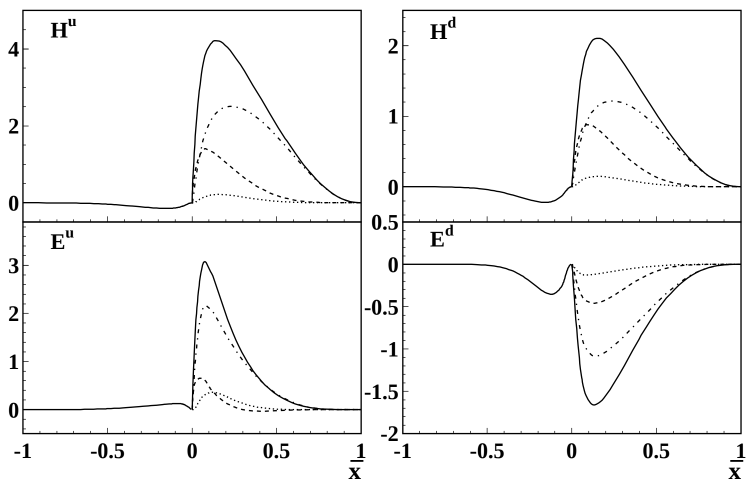


Figure 2. The different contributions to the spin-averaged ($H^q$, upper panels) and helicity-flip ($E^q$, lower panels) generalized parton distributions calculated in the meson-cloud model for flavours $u$ (left panels) and $d$ (right panels), at $\xi = 0$ and $t = -0.2$ GeV$^2$. Line style as in Fig. 1.

at $t = -0.2$ GeV$^2$. The relative contribution of the different components is not modified by switching on the momentum transfer $t$, only the overall magnitude is decreased. This is in agreement with the common believe that the main part of the $t$ dependence of the GPDs is exhibited by their first moments, i.e. by the quark Dirac and Pauli form factors.

In Fig. 3 are plotted the isoscalar and isovector combinations of GPDs plotted at $\xi = 0.1$ and $t = -0.2$ GeV$^2$. We see that GPDs in the ERBL region are rather regular functions over the whole range, with zeros at the endpoints $\overline{x} = \pm\xi$. This result is quite different from the oscillatory behaviour predicted by the chiral quark-soliton model [ 5] where the valence contribution of the discrete level is a smooth function extending into the ERBL region and adding to the sea contribution. Here this is forbidden because the support of the valence contribution is limited to the DGLAP region. In addition, the transition amplitude between the bare-proton and the baryon-meson components vanishes at $\overline{x} = \pm\xi$, approaching these points both from the ERBL and DGLAP region. This generates a discontinuity of the first derivative of GPDs at $\overline{x} = \pm\xi$ which, however, is not in contradiction with general principles [ 6].

Furthermore, in the DGLAP region both for $\overline{x} \geq \xi$ and $\overline{x} \leq -\xi$ no striking difference arises in Fig. 3 for the spin-averaged GPDs $H^{u\pm d}$ with respect to the results in the forward limit shown in Fig. 1, while for the helicity-flip GPDs the (negative) $d$ contribution coming from the baryon in the $|BM\rangle$ component is responsible for a broader shape at $\overline{x} \geq \xi$.

## 4. Conclusions

We described a convolution model for the unpolarized GPDs where the nucleon is viewed by a quark-valence core surrounded by a mesonic cloud. This meson-cloud model gives the

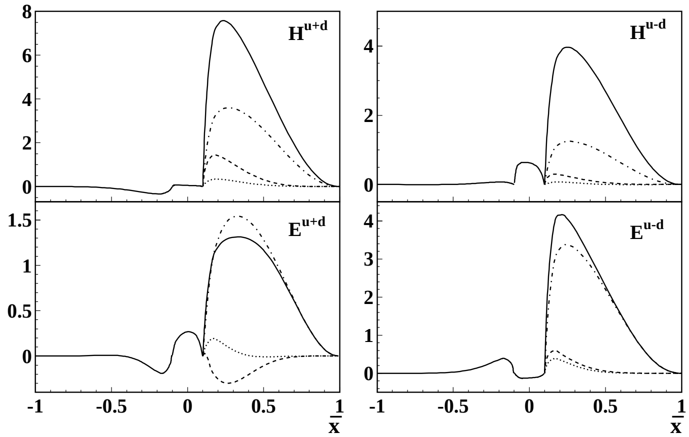


Figure 3. Isoscalar ($u+d$, left panels) and isovector ($u-d$, left panels) combinations of the spin-averaged (upper panels) and helicity-flip (lower panels) generalized parton distributions calculated in the meson-cloud model, at $\xi = 0.1$ and $t = -0.2$ GeV$^2$. Line style as in Fig. 1.

possibility to link GPDs calculated in the light-front formalism to the nucleon description in terms of constituent quarks including a sea contribution already at a low-energy scale. Since the contribution in the ERBL region is vanishing in the forward limit, it can not be easily inferred from parametrizations in terms of parton distributions. Therefore, the present calculation gives new insights to model the off-forward features of the GPDs, and can be further used as a suitable input at the hadronic scale to study the behaviour under evolution at higher scales.

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